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QUANTUM INFORMATION

Source of triggered entangled photon pairs?

Arising from: R. M. Stevenson *et al.* *Nature* **439,** 179–182 (2006)

The realization of an entangled photon source will be of great importance in quantum information — for example, for quantum key distribution and quantum computation — and Stevenson *et al.*[1] have described such a source. However, we show here that first, their source is not entangled; second, they use inappropriate entanglement indicators that rely on assumptions invalidated by their data; and third, their source has insignificant entanglement even after simulating subtraction of the significant quantity of background noise. We therefore find that the standard of proof required for a semiconductor source of triggered entangled photon pairs has not been met by Stevenson *et al.*[1].

First, Stevenson *et al.*[1] produce pairs of photons by radiative decay of biexcitons in single quantum dots, and measure strong coherences between two polarization populations, a necessary but not sufficient condition for entanglement. Establishing the degree[2–5] or presence[6] of entanglement between two qubits is a solved problem: Stevenson *et al.*[1] apply none of these standard techniques. The entanglement of formation[2], E_F; concurrence[3,4], C; and tangle[5], T, are directly related (for example, $C = \sqrt{T}$), and are quantitative, measuring the degree of entanglement. Other methods, such as the Peres condition[6], or fidelity with a maximally entangled state, are qualitative, indicating but not measuring entanglement. Any of these quantities can be calculated directly from two-photon density matrices obtained by quantum state tomography[7], as measured in Fig. 3 of ref. 1.

For example, using the tangle to quantify unambiguously the degree of entanglement ($0 \leq T \leq 1$; T=0 for unentangled states and T=1 for maximally entangled states[4]), we find that T=0 for every measured density matrix[1]. There is no entanglement. The large observed coherences in dots B and C in Fig. 3b,d of ref. 1 indicate only classical correlation. State purity provides an independent check. Quantified by the linear entropy ($0 \leq S_L \leq 1$; $S_L = 0$ for pure states and $S_L = 1$ for maximally mixed states[5]), states with $8/9 < S_L$ always have strictly zero entanglement[8]. The states in Fig. 3 of ref. 1 are highly mixed, with $0.92 \leq S_L \leq 0.99$, and so are unentangled. Similarly, all other metrics and indicators listed here find no entanglement.

Second, Fig. 2c of Stevenson *et al.*[1] measures the degree of correlation for quantum dots A and B. Entanglement is indicated by a constant value that is greater than |50|% (ref. 9). The authors arbitrarily fit a sinusoid to dot A, and a constant value to dot B (22.2 ± 2.8%). (There are no statistical grounds for asserting that dot B's correlation is independent of measurement basis[1]. Dot B is better fitted with a sinusoid with the same period as dot A, which ranges from 20% to 25%: $\overline{\chi}^2 = 0.065$ compared with $\overline{\chi}^2 = 0.13$.) Irrespective of curve choice, these data provide no evidence for entanglement, as the mean for dot B is 9.9σ less than 50%. The curve shape is a very poor indicator of entanglement, because it varies greatly even between different unentangled sources: dots A and B are both unentangled, albeit with different classical correlations.

Stevenson *et al.*[1] also use an eigenvalue entanglement indicator. Only if both individual photons are unpolarized is entanglement indicated by the largest eigenvalue of the two-photon density matrix exceeding 0.5. The only data of Stevenson *et al.*[1] that provide a direct measure of the polarization of the photons emitted by the dots are the density matrices in their Fig. 3. These are a precise measure and are self-consistent: the same data used by Stevenson *et al.*[1] to gauge entanglement can also be used to check the photon polarization. For $\boldsymbol{\rho}_{3d}$, we obtain the polarization of each photon in the pair by tracing out the other, finding that one is partially polarized with a degree of polarization[10,11] of 4.5 ± 1.9% — the eigenvalue method is therefore invalid. (Degree of polarization is the length of the Stokes vector[10], $\wp = (1 - 4\,|\boldsymbol{\rho}_i|)^{1/2}$, in which $|\boldsymbol{\sigma}_i|$ is the determinant of the single-polarization density matrix, $\boldsymbol{\rho}_i$ (ref. 11).) Note that the other photon has zero polarization, 0 ± 1.1%, owing to the artificial normalization imposed by Stevenson *et al.*[1] (Table 1).

Based on the data in their Figs 2 and 3, Stevenson *et al.*[1] claim that their measurements indicate that dots with small exciton splitting emit entangled photons. As we have shown, this conclusion is not supported by their data or methods.

Table 1 | **Density matrix of Fig. 3d of ref. 1 and the underlying probabilities***

Measure	P	ΔP	Measure	P	ΔP	Measure	P	ΔP	Measure	P	ΔP
VV	0.30470	0.00606	LV	0.25592	0.00737	DL	0.23479	0.00919	HD	0.24041	0.01065
VH	0.19530	0.00496	LH	0.24408	0.00562	DD	0.30285	0.00463	HR	0.23647	0.01235
HH	0.32605	0.01084	DH	0.25058	0.00459	LD	0.26214	0.00631	VR	0.27020	0.00773
HV	0.17395	0.01409	DV	0.24942	0.00651	VD	0.26054	0.00511	LR	0.30477	0.00769

*R. M. Stevenson *et al.*, personal communication.

Measure refers to polarization analyser settings for each photon: V, vertical linear; L, left circular; D, diagonal linear; H, horizontal linear; R, right circular. Standard unbiased tomography uses complete measurement sets to ensure that the sum of the probabilities is unity — for example, $P_{VV} + P_{VH} + P_{HV} + P_{HH} = 1$. Repeating this in each measurement basis allows for compensation of measurement drift. This is not done by Stevenson *et al.*[1], who instead use an artificial normalization that sets the sum of each of their first four probability pairs to a half — for example, $P_{VV} + P_{VH} = 0.5$. The effect of their extra constraint is to force one photon in each pair to be unpolarized. The resulting density matrix, $\boldsymbol{\rho}_{3d}$, is accordingly biased:

$$\boldsymbol{\rho}_{3d} = \begin{bmatrix} 0.3261 & -0.0096 & -0.0101 & 0.1038 \\ -0.0096 & 0.1739 & 0.0009 & 0.0101 \\ -0.0101 & 0.0009 & 0.1953 & 0.0105 \\ 0.1038 & 0.0101 & 0.0105 & 0.3047 \end{bmatrix} + i \begin{bmatrix} 0 & -0.0135 & 0.0166 & 0.0002 \\ 0.0135 & 0 & -0.0235 & -0.0166 \\ -0.0166 & 0.0235 & 0 & 0.0202 \\ -0.0002 & 0.0166 & -0.0202 & 0 \end{bmatrix}.$$

Methods. To perform unbiased quantum state tomography and a full uncertainty analysis, we need count data for the probabilities, P_i, in Table 1. Stevenson *et al.*[1] use second-order correlations, $g^{(2)}(\tau = 0) = C/(N/n)$, between photons 1 and 2, to obtain $P = 1/2\, g_A^{(2)} / (g_A^{(2)} + g_B^{(2)})$, where A and B refer to orthogonal measurements on photon 2; C is the number of pairs detected in the same laser cycle; N/n is the average number of pairs in different laser cycles; n is the number of measured finite delay peaks; the uncertainty is $\Delta g^{(2)} = g^{(2)} (C^{-1} + N^{-1})^{1/2}$, assuming $\Delta n = 0$ and poissonian uncertainties, $\Delta C = \sqrt{C}$, $\Delta N = \sqrt{N}$ (R. M. Stevenson *et al.,* personal communication). The count data, C, N or n, was not available, the figure of 1,000 coincidences given in ref. 1 was simply provided as a guideline (R. M. Stevenson *et al.,* personal communication). Accordingly, it is not clear how ΔP_i were obtained: for N counts, poissonian uncertainties are $\sqrt{N}$, and cannot be calculated from a probability. In the absence of counts, we used the artificially normalized probabilities to reconstruct $\boldsymbol{\rho}_{3d}$; for all other density matrices, we estimated the values of the elements directly from Fig. 3 of ref. 1. Linear entropy and tangle are calculated directly from the reconstructed/estimated density matrices[3–5,7]. We estimated uncertainties from an ensemble of 5,000 density matrices generated by creating a new data set by sampling from a gaussian distribution centred on P_i, with standard deviation equal to ΔP_i, and by applying maximum-likelihood tomography to each such data set. The tangle and degrees of polarization were calculated for each of the 5,000 matrices; uncertainties are the standard deviation of these quantities. To model background subtraction, 5,000 physical density matrices were obtained after subtracting 0.49 $\boldsymbol{I}/4$, where $\boldsymbol{I}$ is the 4×4 identity matrix.

Third, unwanted background light degrades entanglement. Stevenson *et al.*[1] identify 49% (no error given) of photon pairs as background due to dark counts and emission from layers other than the dot. An improved source was simulated by subtracting the projected (but not directly measured) number of background counts; the resulting density matrices and data are not given. Accordingly, we modelled this by subtracting 49% unpolarized light from ρ_{3d}: the simulated source has insignificant tangle, $T = 0.028 \pm 0.022$. Naturally, removing unpolarized light serves to increase further the polarization of the partially polarized photon, to 8.8 ± 3.4% — the eigenvalue method remains invalid. Appropriate background subtraction[12] may indicate the potential of an entangled source, but such a source is not useful in quantum information. For example, the security of the Ekert protocol in quantum cryptography requires actual, as opposed to virtual, entanglement.

Alexei Gilchrist*, Kevin J. Resch*†, Andrew G. White*
*Department of Physics and Centre for Quantum Computer Technology, University of Queensland, Brisbane, Queensland 4072, Australia
e-mail: andrew.white@physics.uq.edu.au
†Present address: Department of Physics and Institute for Quantum Computing, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

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Stevenson et al. *reply*

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We have reported triggered photon-pair emission from single quantum dots that is suggestive of polarization entanglement[1]. Gilchrist *et al.*[2] criticize our analysis of the density matrix, claiming that the entanglement test is inappropriate. However, we show here that this analysis is in fact valid for an unpolarized source such as ours, and that this analysis (as well as alternative measures of entanglement) suggests that quantum-dot sources do emit entangled photons, and that their quality is rapidly improving[3].

Gilchrist *et al.*[2] disregard the fact that direct measurement of the dot emission shows the source to be unpolarized, within error (Fig. 1). Measuring linearly polarized exciton and biexciton intensities as a function of rotation of a half-wave plate yields polarizations of 0 ± 0.5% and 0 ± 1.1%, respectively. Our use of the eigenvalue test of entanglement is therefore valid. A less precise, although consistent, polarization measure is determined from the photon-pair intensities shown by the two-photon density matrix in Fig. 3d of ref. 1. From this density matrix, we determine polarizations of 0 ± 4.6% and 4.5 ± 4.6% for the two photons, confirming independently that emission is unpolarized.

The eigenvalue is an intuitive test for entanglement and is simply the probability that the source emits into a single state, which naturally cannot exceed 50% for a classical source that has at least one unpolarized photon. We analysed light originating from the dot alone by subtracting contributions from other layers in the sample determined directly from the emission spectrum of the source[1]. Background correction is a well established procedure[4] and it is reasonable to assume that dot emissions can be isolated better with improved design of devices[3]. We pointed out[1] that only after correction did the measured eigenvalue of 0.58 ± 0.04 violate the 0.5 limit for an unpolarized classical source. The error on the background level of <1% makes only a minor contribution to this error in the eigenvalue. For an unpolarized, partially uncorrelated classical source[1], the maximum possible eigenvalue is only 0.4.

Using data supplied by us, Gilchrist *et al.*[2] calculate tangle $T = 0.028 \pm 0.022$ and concurrence $C = \sqrt{T} = 0.167 \pm 0.090$. The values of $T > 0$ and $C > 0$ show that emission from the dot must contain entangled photons[2], in general agreement with our results[1].

Although we show that photon pairs originating from a quantum dot violate classical limits, our most important message[1] is the idea of how to manipulate quantum dots to generate entangled photons. By selecting dots with specific emission energy, or applying an appropriate magnetic field, the exciton polari-

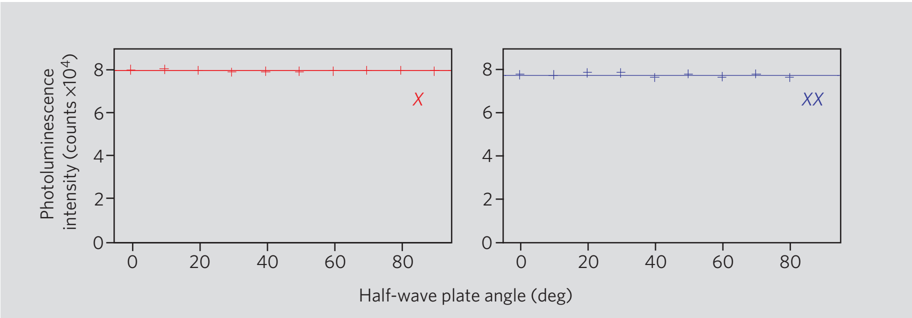

Figure 1 | Measurements showing that emission is unpolarized, within error. Linearly polarized exciton (X) and biexciton (XX) intensities were measured as a function of the angle of a half-wave plate. Intensities are independent of the wave-plate angle and thus of the linear polarization detection basis. The degree of polarization is therefore zero, with errors of 0.5 for the exciton photon and 1.1% for the biexciton photon. Note that the biexciton error is higher owing to greater sensitivity to excitation power fluctuations. A similar result is obtained for circular polarization. The probability of detecting polarized photon pair combinations was measured experimentally from pairs of second-order correlations with orthogonal exciton polarizations as described in ref. 1; that is, $P = 0.5\, g^{(2)}_{x,xx}/(g^{(2)}_{x,xx} + g^{(2)}_{x,\bar{x}\bar{x}})$. Corresponding errors are determined by standard analysis of the correlation function errors. Because the emission is unpolarized, this normalization compensates for excitation drift and allows formation of a density matrix for the emission; 16 such probability measurements were recorded to construct the density matrix shown in Fig. 3d of ref.1 (see also ref. 2). The degree of polarization is the difference between orthogonally polarized photon intensities, divided by the sum. The intensity of a polarized photon is determined from a density matrix by the sum probability of detection with another photon of equal or opposite polarization. Errors are determined by propagating the measurement errors used to construct the matrix. The polarization assessments from rectilinear, diagonal and circular bases are combined and errors compounded, to give an overall polarization with error.