Quantum information and Optics

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This is the first of two articles that look at the new field of *quantum information* and it's relationship with optics. In this article we introduce the central concepts of quantum information, illustrating them with simple optical examples. In the next article we will look at making entangled photons, and some of their recent applications, including tests of nonlocality, quantum cryptography, and quantum computation.

Flicking through Physical Review the last couple of years you may have noticed a new section heading - *quantum information* (QI). And, like many colleagues of mine, you may have read an article or two in that section but were put off by the theory, or worse, the jargon, and not read on. If so, fear not because: a) you're not alone and b) many of the key ideas of quantum information are particularly accessible via optics, and so to readers of AOS News.

So what is quantum information? A standard definition[1] is "…the application of quantum mechanics to information theory…", which while succinct, isn't terribly informative. Perhaps better is the slogan "no information without representation", which highlights a key concept: all information storage & processing is achieved via some *actual* system, and the physics (and indeed chemistry, biology, etc.) of that system necessarily constrains the storage & processing.

If it were only a matter of constraints then our slogan is really not very exciting. However, if we treat the physical constraints as *fundamental*, and explore the consequences for information processing given these fundamental limits (particularly quantum mechanical limits) we come to a powerful realisation. It is possible to achieve information processes in quantum mechanical systems that are *impossible* with classical systems (and classical computational logic).

"Enough!" I hear you cry (or that may be someone else, in which case, thank you for your patience) "…what about optics?" In this and the next article we look at some of the basic concepts in quantum information, illustrating them with optical examples, and look at some recent optical QI experiments. On the way we'll answer important questions including: *what are qubits? what is entanglement? are there different kinds of entanglement? how is it characterised? why is it powerful? how is it measured?* …and most importantly of all … *why should I care?*

[1] "Standard" in that I bandied it around the department and noone disagreed.

I. AN INTRODUCTION TO QUANTUM INFORMATION

A. What are qubits?

A qubit, or quantum bit, is *any* two level quantum system. Figure 1 shows some common examples: spin (electronic or nuclear); polarisation of light; energy levels in an atom, ion, quantum dot, nucleus, *etc.* In this article we will concentrate on using the polarisation of light as a qubit. Like classical bits, qubits can exist as a 0 or a 1; unlike classical bits, they can exist in superposition states. If we consider horizontal polarisation, $|\mathrm{H}\rangle$, as our logical "0", and vertical polarisation, $|\mathrm{V}\rangle$, as our logical "1", then it becomes clear that diagonal polarisation, $|\mathrm{D}\rangle = \frac{1}{\sqrt{2}}\,(|\mathrm{H}\rangle + |\mathrm{V}\rangle)$, is the logical state $\frac{1}{\sqrt{2}}\,(0+1)$, and right-circular polarisation, $|\mathrm{R}\rangle = \frac{1}{\sqrt{2}}\,(|\mathrm{H}\rangle + \mathrm{i}|\mathrm{V}\rangle)$, is the logical state $\frac{1}{\sqrt{2}}\,(0+i1)$. Note that in quantum information the transformation $0 \to \frac{1}{\sqrt{2}}\,(0+1)$ is called the *Hadamard transform* - in polarisation optics we call a device that does this a *waveplate* (e.g., $|\mathrm{H}\rangle \to 1/\sqrt{2}\,(|\mathrm{H}\rangle + |\mathrm{V}\rangle)$).

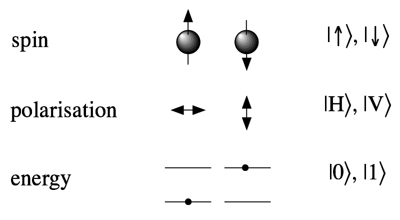

FIG. 1: Examples of qubits: a) spin b) polarisation c) energy levels.

Qubits carry phase information. For example, diagonally- and right- circularly polarised light both are equal weight superpositions of horizontal and vertical light - they differ only in their relative phase. Experimentally, if we send either a diagonal- or right- circular photon onto a polarising beamsplitter, it appears at the horizontal or vertical output ports with a probability of 50%, as shown in Figure 2a. This is no different to a 50/50 mixture of classical bits. However, as shown in

Figure 2b, if we take the outputs of the beamsplitter and combine them onto a second polarising beamsplitter (with equal path lengths), we recover the original polarisation state and the qubit will always pass an analyser set at $\theta°$. This kind of process is impossible with classical bits.

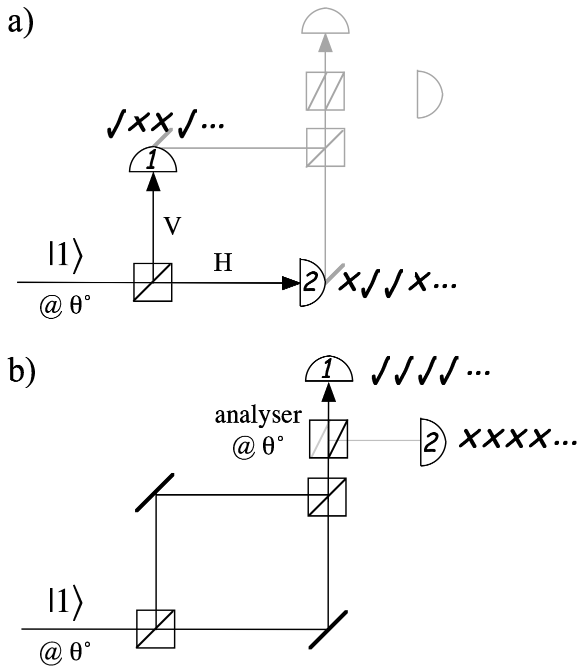

FIG. 2: Qubits carry phase information. a) a polarisation qubit, $\theta°$, incident on a polarising beamsplitter. b) a polarisation qubit incident on a polarising interferometer. The qubit is passed unscathed by the interferometer.

B. What is entanglement?

So is the only difference between classical and quantum information the fact that in the latter we use qubits, and these can be superpositions? The answer is no, there is one other key difference: qubits can be correlated in a way that can not be mimicked using classical bits. This "super correlation" is known as *entanglement*[2] after Schrödinger [1]. So what is entanglement? Textbooks normally start with a mathematical definition, but we are going to eschew that for an optical example.

Let Alice and Bob be two individuals with too much time on their hands. There is an unknown source of light sending photons to both Alice and Bob, as shown in Figure 3. They wish to determine the polarisation properties of the light. Alice analyses only in the H/V basis, using some polariser (where $0° \equiv$ H, $90° \equiv$ V). In either basis, half the time she sees a photon, i.e $P_{\rm H} = P_{\rm V} = \frac{1}{2}$. Bob, meantime, is more of a free spirit, randomly analysing in many bases, θ (where $0° < \theta < 180°$, of course). Bob finds that the light appears totally unpolarised, consistent with Alice's observation.

In addition, both Alice and Bob keep a timing list, as follows: *Alice* "At 1 ns, I saw a photon at 0°; at 2 ns, I saw nothing; at 3 ns, I saw a photon at 0°. . . " *Bob* "At 1 ns, I saw nothing; at 2 ns, I saw a photon at 12°; at 3 ns, I saw a photon at 47°. . . "

After doing this for a while, Alice and Bob stop, and get together in the pub (why not?) to compare lists. In particular, they calculate the probability of Bob seeing a photon when Alice sees a photon - the *coincidence proabability*, P_{AB}. They make the interesting observation that whenever Alice saw a photon at 0°, Bob never saw a photon at 90°, i.e. $P_{HV} = 0$, and in fact, had a perfect probability of seeing a photon at 0°, $P_{HH} = 1$. The coincidence probability versus Bob's analyser setting looks like that given in Figure 4.

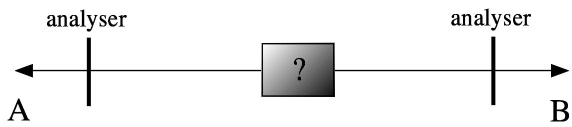

FIG. 3: An unknown source of light, analysed by Alice (A) and Bob (B).

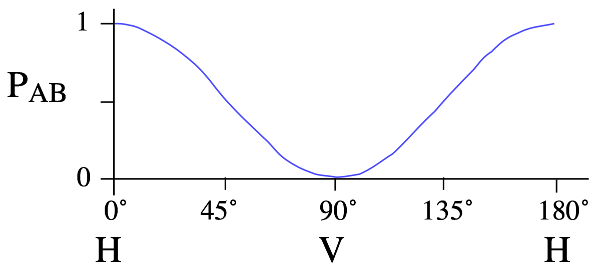

FIG. 4: Coincidence probability vs Bob's analyser setting. Alice is analysing in the H/V (0°/90°) basis.

So what is this source? One possibility is a random mixture of pairs of horizontally and vertically polarised photons. For example: at 1 ns, a pair of horizontally polarised photons might be sent to Alice and Bob; at 2 ns, another horizontal pair; at 3 ns a vertical pair, and so on. To check this possibility, Alice and Bob return the lab (fortified by fine ale) and repeat their measurements, with the only change being that now Alice analyses in the diagonal/anti-diagonal basis (45°/135°). Again, after a period of time, they stop measuring and repair to the pub. Now they observe that whenever Alice saw a photon at 45°, Bob never saw a photon at 135°, $P_{D\bar{D}} = 0$, and the coincidence probability versus Bob's analyser setting looks like that given in Figure 5.

[2] Although perhaps a better, and certainly more euphonious, translation from the German would be "entwinement".

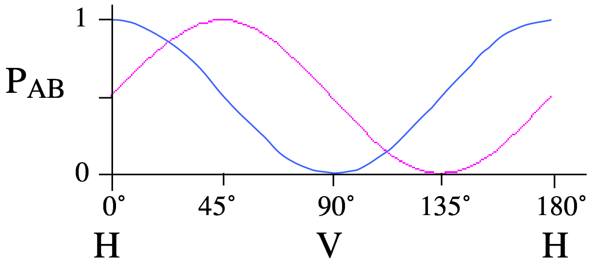

FIG. 5: Coincidence probability vs Bob's analyser setting. Alice is analysing in the D/D̄ (45°/135°) basis.

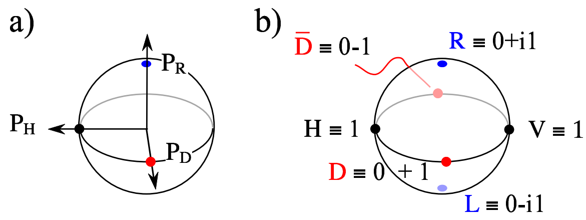

FIG. 6: Poincare sphere. a) a set of measurement axes that define the sphere. b) positions of a range of polarisation (logical) states on the sphere.

This means the source cannot be a random mix of horizontal and vertical pairs. If it were, then Alice may see a diagonal photon, with $P_D = \frac{1}{2}$, and Bob may see an anti-diagonal photon, with $P_{\bar{D}} = \frac{1}{2}$, and the coincidence probability would be measured to be $P_{D\bar{D}} = \frac{1}{4}$, not $P_{D\bar{D}} = 0$. So what is the source? To further confound our heroes, they find that regardless of which basis Alice chooses for her measurements, Alice and Bob always find a perfect visibility coincidence fringe, i.e. they observe *perfect correlations (or anti-correlations) in every measurement basis*. This is entanglement.

In quantum information terms, Alice and Bob have a source of perfectly entangled qubits. How many bases do they need to measure in before they completely characterise the entanglement? Two? Infinity? . . .

C. Characterising qubits and systems of qubits

In general, 3 parameters are required to completely characterise a qubit. The qubit can be represented graphically by using these parameters to plot its position on, or in, some characteristic sphere. For polarisation, this is the Poincaré sphere (for spin, the Bloch sphere). In the Poincaré sphere, as shown in Figure 6, the axes indicate measurement probabilities in some appropriate basis set, e.g. H, D, & R. The polarisation of any light source can be mapped onto the sphere by simply measuring the probabilities of the light passing through H, D, & R polarisers, respectively. Note that this requires 4 intensity/count rate measurements: I_H, I_D, & I_R plus, say, I_V, to give the total intensity/count rate ($I_0 = I_H + I_V$) and enable normalisation (e.g. $P_D = I_D/I_0$). These measurements are also known as the *Stokes parameters*, where $S_0 = I_0$ and $S_{1,2,3} = I_{\mathrm{H,D,R}}$, and are related by $S_0^2 = S_1^2 + S_2^2 + S_3^2$ [2].

Completely polarised light will lie on the surface of the sphere. In quantum mechanical terms we say this is a *pure state*, and note that it is highly ordered. If the measured light lies at the centre of the sphere, i.e. has equal probability, $P = \frac{1}{2}$, of being found in any basis, the light is unpolarised. In quantum mechanical terms we say the state is *mixed*, and it is highly disordered. If the state lies between the centre and the surface of the sphere, the light is partially polarised (partially pure). It is always possible to uniquely decompose a single qubit into a pure and a mixed component (or in polarisation terms, into completely polarised and unpolarised components.)

Alternatively, qubits can be represented by a characteristic matrix. For polarisation, this is the *coherency matrix*, or in quantum mechanical terms, the *density matrix*, $\hat{\rho}$. In such matrices, the the diagonal elements are populations, and the off-diagonal elements are coherences. A range of typical matrices, and the states they represent, are shown below. The density matrix contains *all* the information about the qubit. For example: the purity, or degree of polarisation, of the qubit is $\mathcal{P}$=Tr$\{\hat{\rho}^2\}$; the von Neumann entropy is $\mathcal{S}$=-Tr$\{\hat{\rho}\log_2(\hat{\rho})\}$; and the normalised linear entropy is $\mathcal{S}_L$=$2(1-\mathcal{P})$.

qubit state	ρ	ρ	purity
H-polarised light (0°)	$\begin{matrix} & \langle H\vert & \langle V\vert \\ \vert H\rangle & 1 & 0 \\ \vert V\rangle & 0 & 0 \end{matrix}$	$\vert H\rangle\langle H\vert$	1
D-polarised light (45°)	$1/4 \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\vert H\rangle\langle H\vert+\vert H\rangle\langle V\vert$ $+\vert V\rangle\langle H\vert+\vert V\rangle\langle V\vert$	1
unpolarised light (-°)	$1/2 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\vert H\rangle\langle H\vert+\vert V\rangle\langle V\vert$	0

FIG. 7: Single qubit density matrices

What about a system of qubits? We've already consisted the simplest multi-qubit system in the previous section: two entangled qubits. It turns out that such a system cannot be completely characterised by isolated measurements on it's subsystem:, coincident measurements are required. In polarisation terms, it is no longer enough to measure the Stokes parameters of each beam: we need to measure the *bi-photon Stokes parameters* [3, 4]. These are 16 coincidence measurements, one possible set being the pairwise combination of the traditional Stokes parameters, i.e. HH, HD, HR, HV; DH, DD, DR, DV; RH, RD, RR, RV; VH, VD, VR, VV. While it is somewhat difficult to draw a 15-dimensional sphere that represents the entangled state, it is relatively straightforward to combine these measurements into a 4x4 density matrix. Some experimentally measured density matrices, and the states they represent, are shown in Figure 8.

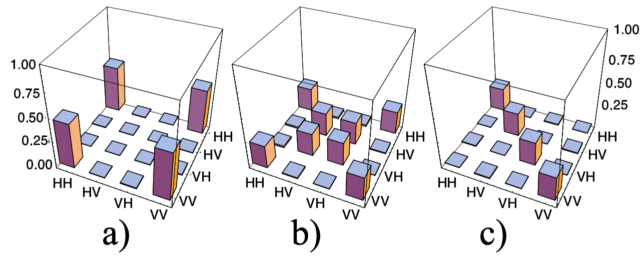

FIG. 8: Two qubit density matrices (measured). a) the maximally entangled state, $\frac{1}{\sqrt{2}}(|\mathrm{HH}\rangle + |\mathrm{VV}\rangle)$. b) a half-mixed, unentangled state. c) the fully mixed state.

D. Power of entanglement

Why is entanglement seen as a desirable characteristic in quantum information? A full answer to this is beyond the scope of this article but we can begin to get an insight by considering the number of parameters required to characterise a system of qubits.

As we've seen, a single qubit can be described by 3 parameters. If we have N qubits, but the qubits are not entangled, then each qubit can be described by 3 parameters and the total number of parameters required to describe the system is simply 3N (e.g. the polarisation of N separate laser beams).

However, if the N qubits are entangled, then the system is described by a density matrix of dimension, d = 2^N, which in turn requires $2^{\mathrm{d}} - 1 = 4^{\mathrm{N}} - 1$ parameters to describe (or $4^{\mathrm{N}} - 1$ measurements). Clearly, there is an exponential blow-out in the number of parameters required to describe the system, as shown in the Table below. From an experimental point of view, this means that even a system of just 5 entangled qubits requires 1024 measurements (including the normalisation) - it rapidly becomes a laborious task to fully characterise the system.

Instead of asking how many parameters are required to describe the state, quantum computation inverts the problem and treats that number of parameters as computational degrees of freedom, with only one measurement made at the output of the device. So, in effect, the number of parameters are proportional to the computational power, and that number increases exponentially with the number of entangled qubits. Although this is a major over-simplification (e.g., it assumes mixture is computationally useful) it does capture the flavour of why quantum computation, and entanglement, is so powerful.

TABLE I: Scaling with number of qubits, N. $\hat{\rho}$ is the density matrix that describes a N-qubit system.

# of qubits, N	dimension of $\hat{\rho}$, 2^N	# of parameters, $4^N - 1$
1	2	3
2	4	15
3	8	63
4	16	255
5	32	1023
⋮	⋮	⋮

E. Characterising entanglement

Let us return to just two entangled qubits. There are a wide range of entangled states - however just 4 states suffice to form a basis that span the space of possible states. These are the *Bell states*: in polarisation terms, $|\phi^{\pm}\rangle = |\mathrm{HH}\rangle \pm |\mathrm{VV}\rangle$ & $|\psi^{\pm}\rangle = |\mathrm{HV}\rangle \pm |\mathrm{VH}\rangle$. (Note that the normalisations have been omitted here, as they often are in quantum information articles, but they are very important!) For all 4 Bell states, the correlations are perfect, but they look different in different bases. For example, $P_{HH} = 1$ for the $|\phi\rangle$ states, whereas $P_{HH} = 0$ for the $|\psi\rangle$ states. More subtly, $P_{DD} = 1$ for $|\phi^{+}\rangle$, but $P_{DD} = 0$ for $|\phi^{-}\rangle$, and so on.

With 16 measurements (given by the bi-photon Stokes parameters) we can reconstruct the density matrix and completely characterise the state of two qubits. We can then analyse this in a number of ways. Perhaps the simplest is to look at the overlap, or fidelity, between the measured density matrix and some ideal density matrix, as shown in Figure 9.

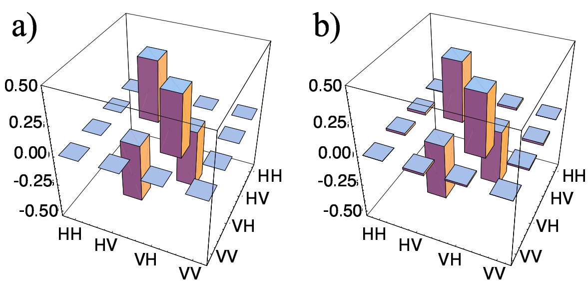

FIG. 9: a) density matrix of an ideal maximally entangled state b) tomographically reconstructed density matrix (the imaginary components are on the order of a few percent, and are not shown). The Fidelity between these matrices is 0.97 ± 0.03 - the measured state is quite entangled.

A more quantitative approach is to analyse both the degree of order and the degree of correlation in the measured density matrix. Earlier we discussed several measures for the degree of order (purity, von Neumann entropy, linear entropy). The degree of correlation can be extracted by calculating either the *entropy of entanglement*, or the *tangle* [5]. Measured entangled states can then be compared to one another by plotting their position on the tangle-entropy plane. Until recently, it has only been possible to produce either highly-entangled, highly-ordered states (circled area, top left, Figure 10), in optical, atomic and ionic systems; or unentangled, highly-disordered states (circled area, bottom right, Figure 10) in liquid-phase NMR systems. It was something of an open question as to what states, if any, were possible outside of these regimes. However, using optical qubits it is possible to controllably vary both the amount of entanglement and order (specifically, vary the degree of polarisation), as Figure 11 shows an entire range of states, covering the T-S plane, have now been made experimentally.

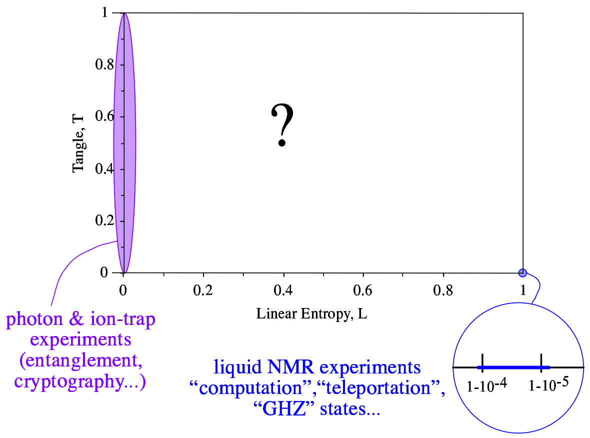

FIG. 10: Location of previous QI experiments on the tangle-entropy plane. The question mark indicates uncertainty over what states, if any, could be produced and characterised outside of these regimes.

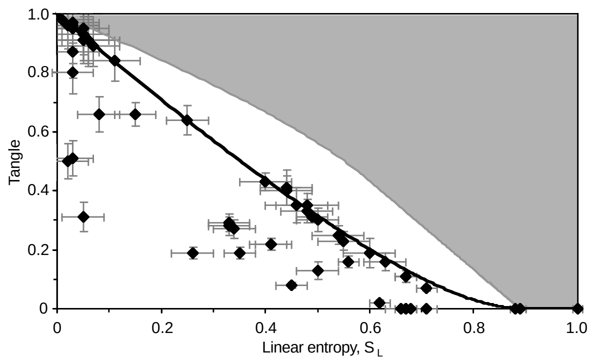

FIG. 11: Location of recent optical 2-qubit states on the tangle-entropy plane. The data points are calculated tangle and linear entropy from a range of measured density matrices. The black curve indicates the *Werner* states: these are states that are a combination of maximally mixed and non-maximally entangled components. The grey region indicates physically impossible combinations of T and S_L.

II. QUANTUM INFORMATION IN AUSTRALIA

If you've read this far, please accept my most hearty congratulations! In the next article we get onto the good stuff: making entangled photons and some of their experimental applications in recent years (from tests of non-locality to quantum computation). Before I leave you, however, let me finish with an update on quantum information research in Australia.

Quantum information can be divided into two major categories: systems of discrete variables (such as polarisation and spin); and systems of continuous variables (such as frequency and quadrature). Both discrete and continuous systems can be realised in optics: this article has concentrated on the former as our research group at UQ concentrates on discrete systems. There is excellent experimental and theoretical work on continuous variable systems done by the group of Dr Ping Koy Lam and Prof. Hans Bachor at ANU: entanglement, cryptography, teleportation and so on can all be realised. It would require yet another entire article (!) to describe these systems in detail - I encourage interested readers to contact the ANU group directly.

Outside of optics, there are major research efforts in quantum information in Australia, perhaps the largest being the Centre for Quantum Computer Technology. This is an Australian multi-university effort (with nodes at the Universities of New South Wales, Melbourne, and Queensland, and a major collaboration with Los Alamos National Laboratory) undertaking research on the fundamental physics and technology of building, at the atomic level, a solid state quantum computer in silicon together with other high potential implementations, including optics. The Centre encompasses major research infrastructure at each of the three nodes, including an extensive semiconductor nanofabrication facility, crystal growth, ion implantation, surface analysis, laser physics, high magnetic fields/low temperatures, and has substantial theoretical support.

Of course, all of these groups are very interested in hearing from motivated undergraduate students wishing to pursue PhD's, or motivated PhD's wishing to pursue postdocs. If you fall into one of these categories, I am sure they would love to hear from you!

III. ACKNOWLEDGMENTS

This article would not have occurred without the heroic efforts (read: threats and encouragement) of the editor, Wayne Rowlands. It is based on a presentation given at the inaugural meeting of the Centre for Quantum Computer Technology at the Blue Mountains, December 1999.

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