Squeezed light from second-harmonic generation: experiment versus theory

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We report excellent quantitative agreement between theoretical predictions and experimental observation of squeezing from a singly resonant second-harmonic-generating crystal. Limitations in the noise suppression imposed by the pump laser are explicitly modeled and confirmed by our measurements.

The theory of squeezed states has been successful in predicting the onset of nonclassical noise properties in nonlinear-optical systems.[1] In particular, the observed noise reduction of the squeezed vacuum produced by an optical parametric oscillator agrees quite well with the theoretical prediction.[2] However, in general, the presence of extraneous noise sources means that quantitative predictions of the magnitude and frequency behavior of squeezing frequently shows only qualitative agreement with experiments.[3]

Recently a new theoretical technique, the cascaded quantum formalism,[4] was developed. This technique permits the noise characteristics of the pump source in a quantum optical experiment to be explicitly modeled. Previously only a select class of idealized pump types could be modeled. To our knowledge no experimental test of this formalism has been made. Also, recently a stable bright source of squeezed light based on second-harmonic generation (SHG) was demonstrated.[5] Unlike in the original SHG experiments,[6] this was a singly resonant system, i.e., only the fundamental light was resonantly enhanced. However, direct comparison of the results with theory was impaired by the presence of pump noise and thermal instabilities.

In this Letter we present experimental results showing bright squeezing, using a system similar to that of Ref. 5. By carefully temperature tuning the crystal and locking the laser frequency to the cavity resonance we were able to avoid thermal instabilities for a wide range of pump powers. We show that, by treating the pump laser and SHG crystal as a single quantum system, excellent quantitative agreement can be obtained between theory and experiment. The predictions of the cascaded quantum formalism are confirmed, thus clearly identifying pump noise as the limiting factor in the noise suppression.

The theoretical model that we used includes a fully quantum-mechanical model of a three-level laser. The output of the laser is coupled into a doubly resonant cavity containing a SHG interaction. The squeezing spectrum of the second-harmonic output beam is calculated, and by making the second-harmonic cavity low finesse we can model the singly resonant experiment. In principle, the results of this Letter could be obtained by use of previous techniques[7] and an experimentally determined intensity spectrum for the pump. The advantage of our approach is that the full spectral quantum noise properties of the pump are modeled as a function of a few parameters. Also, if detunings are present, the full quantum noise information for correct prediction of the squeezing spectra.[8]

Our laser model consists of N three-level atoms interacting with an optical ring cavity mode through the resonant Jaynes–Cummings Hamiltonian

$$\hat{H}_l = i\hbar g(\hat{a}^\dagger \hat{J}_{23}^- - \hat{a}\hat{J}_{23}^+), \tag{1}$$

where circumflexes indicate operators, g is the dipole coupling strength between the atoms and the cavity, $\hat{a}$ and $\hat{a}^\dagger$ are the laser cavity mode annihilation and creation operators, and $\hat{J}_{ij}^-$ and $\hat{J}_{ij}^+$ are the collective Hermitian conjugate lowering and raising operators between the $|i\rangle$th and $|j\rangle$th levels of the lasing atoms (and $\hat{J}_i$ is the collective population operator for the $|i\rangle$th level). Lasing is between the upper two levels of the active atoms. The field phase factors have been absorbed into the definition of the atomic operators. The fundamental cavity mode operators of the SHG cavity, $\hat{b}^\dagger$ and $\hat{b}$, interact with the second-harmonic cavity mode operators, $\hat{c}^\dagger$ and $\hat{c}$, through the Hamiltonian[9]

$$\hat{H}_S = i\hbar \frac{\epsilon}{2} (\hat{b}^{\dagger 2}\hat{c} - \hat{b}^2\hat{c}^\dagger), \tag{2}$$

where ϵ is the coupling constant for the interaction between the two modes through the nonlinear crystal.

Following standard techniques,[10,11] we couple the lasing atoms and cavities to reservoirs and derive a master equation for the reduced density operator $\hat{\rho}$ of the system. The driving of the SHG cavity by the laser is modeled by use of the cascaded quantum system formalism of Carmichael and Gardiner.[4] Included in the laser model are atomic spontaneous emissions from level $|3\rangle$ to level $|2\rangle$ and from level $|2\rangle$ to level $|1\rangle$ at rates γ_{23} and γ_{12}, respectively. Incoherent pumping occurs at a rate Γ. γ_p is the rate of collisional or lattice-induced phase decay of the lasing coherence. The laser cavity damping rate owing to the output port that pumps the SHG cavity is $2\kappa_a$. Additional damping owing to losses occurs at a rate $2\kappa_{al}$. The cavity decay rate for the fundamental mode of the SHG cavity is made up of a contribution from the front mirror ($2\kappa_{bf}$), the back mirror ($2\kappa_{bb}$), and the intracavity losses ($2\kappa_{bl}$). Pumping of the fundamental is through the front mirror. The cavity decay rate for

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the second-harmonic-mode SHG cavity owing to transmission through the back mirror and intracavity losses is $2\kappa_c$. We assume that the front mirror is perfectly reflecting at the second harmonic. The resulting interaction picture master equation is

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = {} & \frac{1}{i\hbar}(\hat{H}_l, \hat{\rho}) + \frac{1}{i\hbar}(\hat{H}_s, \hat{\rho}) \\ & + \frac{1}{2}(\gamma_{12}L_{12} + \gamma_{23}L_{23})\hat{\rho} + \frac{\Gamma}{2}(L_{13}\rho)^\dagger \\ & + \frac{1}{4}\gamma_p[2(\hat{J}_3 - \hat{J}_2)\hat{\rho}(\hat{J}_3 - \hat{J}_2) \\ & - (\hat{J}_3 - \hat{J}_2)^2\hat{\rho} - \hat{\rho}(\hat{J}_3 - \hat{J}_2)^2] \\ & + (\kappa_a + \kappa_{al})(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) \\ & + (\kappa_{bf} + \kappa_{bb} + \kappa_{bl})(2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^\dagger\hat{b}) \\ & + \kappa_c(2\hat{c}\hat{\rho}\hat{c}^\dagger - \hat{c}^\dagger\hat{c}\hat{\rho} - \hat{\rho}\hat{c}^\dagger\hat{c}) \\ & + 2\sqrt{\kappa_a\kappa_{bf}}(\hat{a}\hat{\rho}\hat{b}^\dagger + \hat{b}\hat{\rho}\hat{a}^\dagger - \hat{\rho}\hat{a}^\dagger\hat{b} - \hat{b}^\dagger\hat{\rho}\hat{a}), \\ L_{ij}\hat{\rho} = {} & (2\hat{J}_{ij}^-\hat{\rho}\hat{J}_{ij}^+ - \hat{J}_{ij}^+\hat{J}_{ij}^-\hat{\rho} - \hat{\rho}\hat{J}_{ij}^+\hat{J}_{ij}^-)\,. \end{aligned} \tag{3}$$

We may obtain the semiclassical equations of motion directly from the master equation by making the approximation of factorizing expectation values. A number of techniques for obtaining the squeezing spectrum from such a master equation are available. We proceed by assuming that the quantum fluctuations are sufficiently small that we can treat them as linear perturbations near the stable semiclassical steady state. This is appropriate for the levels of squeezing obtained. The spectral matrix is defined as the Fourier-transformed matrix of two time-correlation functions of these small quantum perturbations. The calculation of the spectral matrix from the master equation by use of the generalized P representation[12] is standard. Using the input/output formalism of Gardiner and Collett,[13] we are then able to relate the fluctuations of the output fields to those of the internal fields and thus obtain the spectra in terms of the spectral matrix. A detailed description of this method can be found in Ref. 14.

Strictly speaking, the master equation is obtained under the assumption that all the cavities are high finesse, hence validating the mean-field approximation. However, as we shall see, excellent agreement is found with experiment, even though the green cavity in the experiment is low finesse. (One may also note that, in the absence of pump noise, our theory is in agreement with that of Ref. 5, which is explicitly singly resonant.)

The experimental setup is shown in Fig. 1. The monolithic standing-wave doubler is made of $MgO{:}LiNbO_3$ and is 12.5 mm long. The resonator is formed by dielectric mirror coatings of the curved end faces ($R = 14.24$ mm). The coupling mirror has 99.6% reflectivity at the fundamental and ~4% reflectivity at the second harmonic. The back mirror has 99.9% reflectivity at the fundamental and the second harmonic. The rated loss at the fundamental is 0.1% cm. The pump laser is a diode-pumped monolithic Nd:YAG ring laser and produces a steady single-mode beam at 1064 nm. Locking of the fundamental mode was achieved by active control of the laser frequency. A Faraday isolator is used to prevent backreflection into the laser. The crystal is heated to the phase-matching temperature, ~120 °C, and stabilized to within a 0.001 °C temperature range.

The second-harmonic beam leaves through the front face of the crystal and is separated from the reflected fundamental by two dichroic beam splitters. It then enters a balanced detection system that produces summed and differenced photocurrent signals. The summed photocurrent yields the intensity noise of the light, and the differenced photocurrent yields the quantum noise level.[6] The detectors were balanced to within 0.05 dB. The electronic noise floor of the detectors was greater than 8 dB below the quantum noise level, and the overall efficiency of the detection system was measured to be 64 ± 5%.

In Fig. 2 we present experimental and theoretical intensity spectra for this system. The experimental trace has been corrected for electronic noise and the overall second-harmonic detection efficiency and has been normalized to the standard quantum limit. The green output power is 30 mW (which represents an IR-to-green conversion efficiency of ~50%). Excellent agreement is seen between theory and experiment.

The theoretical laser parameters are chosen to be consistent with those for a Nd:YAG ring laser.[15,16] Two laser parameters are fitted. The pump rate (Γ) is chosen to produce the observed relaxation oscillation at 500 kHz. The total laser losses $[2\kappa_{al}/(2\kappa_{al} + 2\kappa_a)]$ are then chosen such that the quantum noise crossover point in the squeezing spectrum coincides with the experimental result. This leads to a theoretical prediction that the noise power of the laser (with

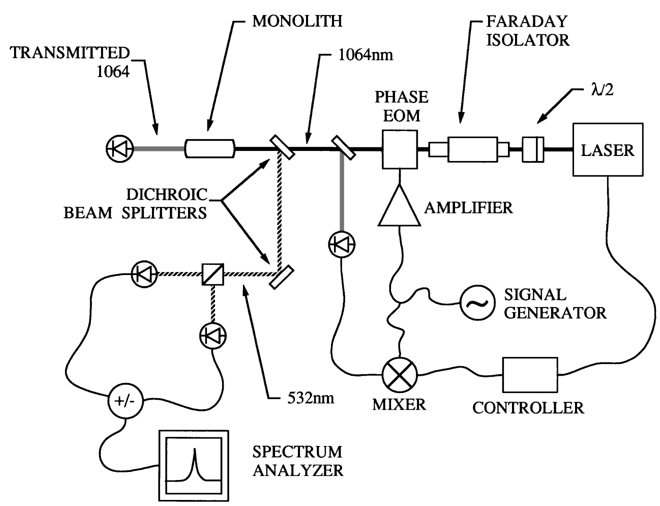

Fig. 1. Schematic representation of the experimental setup. A diode-pumped Nd:YAG laser (Lightwave 122) drives the monolithic doubler. A Faraday isolator prevents significant backreflection entering the laser. A half-wave plate ($\lambda/2$) permits variable attenuation. The electro-optic modulator (EOM) phase modulates the driving field at 88 MHz. An error signal for locking the laser is derived from the 88-MHz signal driving the EOM and the fundamental light reflected from the monolith by use of a double-balanced mixer. The second-harmonic beam exits the front face of the monolith, where it is separated from the fundamental by use of two dichroic beam splitters. It is then incident upon a balanced detector, the summed and differenced outputs of the which are monitored with an HP8568B spectrum analyzer.

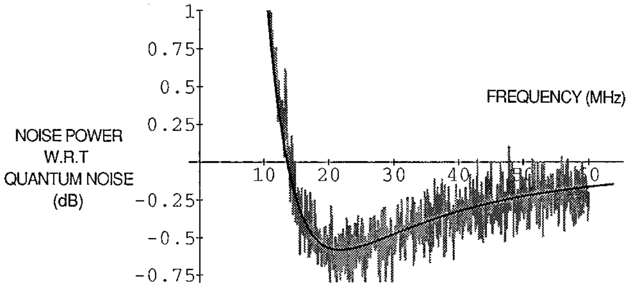

Fig. 2. Comparison of experimental and theoretical noise spectra of the green light produced by the SHG crystal. The experimental traces are obtained with a resolution bandwidth of 300 kHz and a video bandwidth of 100 Hz. They are averages of ten sweeps and have been corrected for electronic noise and the overall detector efficiency. The detected green power is 30 mW. The parameters used to obtain the theoretical traces are $g^2N/\gamma_p = \sigma_s c\rho/2 = 6.7 \times 10^{11}$ s^{-1}, where σ_s is the stimulated emission cross section for Nd:YAG and ρ is the density of Nd atoms; $\gamma_{23} = 4.3 \times 10^3$ s^{-1}; $\gamma_{12} = 3.3 \times 10^7$ s^{-1}; $\kappa_a + \kappa_{al} = 1.4 \times 10^8$ s^{-1}; $\Gamma = 8.3$ s^{-1}; $\epsilon\sqrt{N} = 4.23 \times 10^{12}$ s^{-1}; $\kappa_{bf} = 1.1 \times 10^7$ s^{-1}; $\kappa_{bb} = 3.2 \times 10^6$ s^{-1}; $\kappa_{bl} = 6.7 \times 10^6$ s^{-1}; $\kappa_c = 2.58 \times 10^9$ s^{-1}; and $\kappa_a = 1.54 \times 10^7$ s^{-1}. W.R.T, with respect to.

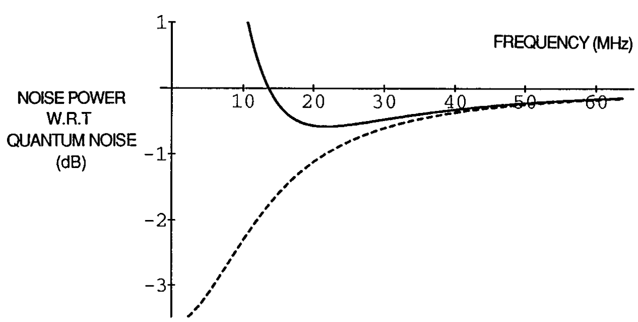

Fig. 3. Comparison of squeezing spectra with (solid curve) and without (dashed curve) pump noise. Parameters are the same as in Fig. 2 except that, for the dashed curve, pumping is achieved by means of a coherent field of equal intensity to the laser output.

~4 mW detected) will come within 0.5 dB above quantum noise at 20 MHz. Comparison of the measured and predicted noise spectra for the high-frequency tail of the laser relaxation oscillation validates this prediction. Notice that squeezing emerges well below 20 MHz. This is because the narrow linewidth of the fundamental cavity acts as a noise filter, reducing the amount of laser noise that enters the crystal. The mirror reflectivities and internal loss of the crystal are known. The interaction strength (ϵ) can be approximately determined from the second-harmonic conversion efficiency.[5] The precise value of ϵ used is a fit to the experimental spectrum at a particular pump power. However, once fitted for one pump power, the theory remains in excellent agreement over the entire currently accessible range of output powers (15–30 mW), correctly predicting the size and frequency of maximum squeezing for different pump powers.

In Fig. 3 we compare the squeezing spectrum obtained under the experimental conditions with that predicted in the absence of laser noise. It is clear from these plots that a major factor limiting the squeezing is the noise of the laser pump. We are currently developing a mode-cleaning system that should decrease laser noise at low frequencies and hence significantly increase the observable squeezing.

In summary, we have presented experimental results showing the generation of 30 mW of approximately 13% (0.6-dB) amplitude-squeezed light from a second-harmonic-generating crystal. We have shown that the experimental results are in excellent agreement with the complete model, which includes the noise of the pump laser explicitly. These results represent what we believe is the first experimental test of the cascaded quantum system formalism. They highlight the limitations imposed by the pump source and permit the theoretical evaluation of potentially better squeezing systems.

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